New Method for Concurrent Dynamic Analysis and Fatigue Damage Prognosis of Bridges

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Abstract: A new methodology for concurrent dynamic analysis and structural fatigue prognosis is proposed in this paper. The proposed methodology is on the basis of a novel small time scale formulation of material fatigue crack growth that calculates the incremental crack growth at any arbitrary time within a loading cycle. It defines the fatigue crack kinetics on the basis of the geometric relationship between the crack tip opening displacement and the instantaneous crack growth rate. The proposed crack growth model can be expressed as a set of first-order differential equations. The structural dynamics analysis and fatigue crack growth model can be expressed as a coupled hierarchical state-space model. The dynamic response (structural level) and the fatigue crack growth (material level) can be solved simultaneously. Several numerical problems with single degree-of-freedom and multiple degree-of-freedom cases are used to show the proposed methodology. Model predictions are validated using coupon testing data from open literature. Following this, the methodology is demonstrated using a steel-girder bridge. The proposed methodology shows that the concurrent structural dynamics and material fatigue crack growth analysis can be achieved.

The cycle-counting method in the conventional fatigue analysis can be avoided. Comparison with experimental data for structural steels and aluminum alloy shows a satisfactory accuracy using the proposed coupled state-space model. DOI: 10.1061/(ASCE)BE.1943-5592.0000227. © 2012 American Society of Civil Engineers.

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Introduction

Metal fatigue is among the most critical modes for bridge safety. The fatigue damage prognosis is still a challenging problem despite tremendous progress made during the past decades. Many fatigue failures have been observed in the United States Japan, and other countries since the 1960s (Miki et al. 2003). Traditionally, bridge fatigue resistance is evaluated by using the S-N curves provided in the AASHTO specifications and the Eurocodes for bridge fatigue design [AASHTO 1992; European Committee for standardization (CEN) 2005]. Miner’s linear damage accumulation rule is the most widely used for fatigue evaluation of bridges because of its simplicity (Miner 1945). But the life prediction on the basis of Miner’s rule is often unsatisfactory for fatigue life under variable amplitude loadings and the influence of fatigue damage on load or global stress response is not considered (Li et al. 2002, 2001). It has been proven by several researchers (Agerskov and Nielsen 1999; Zhao and Haldar 1996) that Miner’s rule is more applicable to the design of new bridges than to evaluation of the fatigue damage and the remaining useful life of existing bridges (Agerskov and Nielsen 1999; Li et al. 2002, 2001; Zhao and Haldar 1996). Several studies have been proposed for bridge fatigue life prediction on the basis of the S-N curve approach (Kwon and Frangopol 2010; Li et al. 2002; Schlaff and Brühweiler 1998). However, the method does not directly associate the fatigue damage with physical mechanisms and cannot provide measurable quantities for the evaluation of the fatigue damage (Li et al. 2002).

The proposed methodology is on the basis of the fracture mechanics, which assumes that the component contains a crack-like defect. This is one of the major hypotheses in the damage tolerance design. There have been a number of researchers performing fatigue damage analysis and fatigue life prediction for larger structures through field measurement. The fatigue failures of Tsing Ma Bridge in Hong Kong were studied by Li et al. (2002, 2001). The fatigue analysis and life prediction was on the basis of obtained data from a structural health monitoring system. Calcade et al. (2002) obtained an experimentally calibrated finite-element model of the bridge on the basis of intensive studies on Luiz I bridge in Lisbon, Portugal. Leander et al. (2010) performed a reliability assessment for a steel railway bridge in Stockholm, Sweden using the field data. However, in these studies, the local information is required from field measurements or a finite-element model before the fatigue analysis can be performed. All these studies used the classical cycle-based fatigue methods and require that the cycle counting of the local loading spectra is done before the fatigue analysis can be performed. The classical fatigue analysis methods separate the mechanical/structural analysis and material damage analysis into two parts. To the best knowledge of the authors, there are no concurrent analysis methodologies that exist for structural response and material fatigue damage. This study aims to develop a general methodology for coupling the structural dynamic analysis and the material fatigue prognosis.

Fatigue damage accumulation is a multiscale phenomenon, which ranges from microstructurally small crack initiation and propagation to structural failure. Classical fatigue prognosis
methods are cycle-based, which correlates the cyclic driving force, e.g., stress or stress intensity factor range, with the number of cycles (Dubey et al. 1997). The earliest work of cycle-based fatigue analysis can be traced back to Wohler (1870) and Basquin (1910) (Schutz 1996). In the 1960s, Paris proposed the fatigue crack growth model that relates the crack growth rate, \( da/dN \), and the stress intensity factor range, \( \Delta K \) [Eq. (1)] (Paris and Erdogan 1963)

\[
da/dN = C\Delta K^m
\]  

where \( C \) and \( m \) = fitting parameters and depend on the applied stress ratio and environment. The Paris’ model makes it possible to predict the remaining useful life using crack growth analysis, which is also the most widely used fracture mechanics—based model. The original Paris model has several limitations. It is only applicable to crack growth rate in the range of approximately \( 10^{-3} \) mm/cycle to \( 10^{-6} \) mm/cycle. The predictions are limited to fixed stress ratios under constant amplitude loading because of both the stress ratio effect and crack closure effect are neglected (Jones et al. 2008; Nguyen et al. 2001; Suresh 1998). Several extensions and modifications of the Paris model have been proposed, such as the inclusion of near threshold crack growth (Laird 1979), crack closure concept (Wolf 1970), stress ratio effect (Gilbert et al. 1995; Gurney 1979), and others. Wolf (1970) observed that fatigue crack surfaces contact under cyclic tensile loading. This observation led to the investigation of the plasticity-induced crack closure concept (Wolf 1970; Newman Jr. and Ruschau 2007). Wolf first established that the fatigue crack propagation is not only influenced by the condition ahead of the crack tip, but also by the premature contact of the crack surfaces behind the crack tip. The fatigue crack remains closed during a part of loading cycles because it propagates in the residual tensile plastic deformed zone ahead of the crack tip (Kujawski 2002). Wolf proposed to use the effective range of the stress intensity factor (SIF) \( \Delta K_{EFF} \) instead of \( \Delta K \) in the Paris model as

\[
\Delta K_{EFF} = K_{max} - K_{OP}
\]

where \( K_{max} \) = maximum stress intensity factor; and \( K_{OP} \) = stress intensity factor at the crack opening load. A considerable number of investigations have been made since Wolf’s discovery (Lee and Song 2000; Solanki et al. 2004).

Another group of models uses the interaction of the plastic zone ahead of the crack tip to explain the different crack growth behavior. For example, fatigue crack growth under a variable loading case has been discussed by Willenborg et al. (1971) and Wheeler (1972). Wheeler postulated that the crack retardation is because of the larger plastic zone ahead of the crack tip that is produced by the overload. A retardation parameter, denoted by \( \phi_R \), is introduced that can be multiplied to any constant amplitude fatigue model. The crack growth under an overload for Paris’ equation is (Willenborg 1971)

\[
\left( \frac{da}{dN} \right)_{\text{retarded}} = \phi_R[C(\Delta K)^m]
\]

The parameter \( \phi_R \) depends on the stress level, the crack shape, and the load spectrums. Wheeler assumed that \( \phi_R \), once calibrated, can be used for other load spectrums. However, it has been shown that the accuracy of prognosis will suffer if different loading spectra are used with the same \( \phi_R \) (Manson and Halford 2006; Pitoniak et al. 1974; Suresh 1983). Both Wheeler’s model and Willenborg’s model are proposed to explain the crack growth retardation induced by overload, but the concept is different. Willenborg determines the amount of retardation as a function of the stress intensity factor necessary to cancel the effect of the overload plastic zone (Taheri et al. 2003). Unlike Wheeler’s model, the Willenborg model does not require an empirical parameter. However, to improve the reliability of the Willenborg model a material constant is introduced (Fatemi and Yang 1998; Stephens and Wei 1976). The acceleration effect that has been observed by many researches (Skorupa 1999; Taheri et al. 2003) has not been included in both models.

The methodologies discussed previously are all cycle based. Although some of these methodologies have been widely used in industry and academia, many inherent difficulties exist for the cycle-based approach. It is impossible to continue reducing the temporal scales because the smallest time scale is one cycle in the cycle-based approach. In addition to the temporal scale limitation, many observed phenomena and research activities are required for the cycle-based model, such as the cycle-counting requirement and mean stress effect in the conventional cycle-based fatigue theory (Gilbert et al. 1995). A detailed discussion is given by Lu and Liu (2010). A real stress time history must be transformed to a cycle history before the fatigue analysis can be performed. This is extremely difficult under the nonproportional multiaxial stress, in which the well-defined cycle history does not exist even under the constant amplitude loading (Kujawski 2002; Lee and Song 2000; Newman Jr. and Ruschau 2007). In addition, this makes the concurrent dynamic analysis and multiscale fatigue prognosis very difficult because the structural/mechanical analysis has to be performed first to get a loading cycle history for the material level damage analysis. In view of this, a fundamentally different and innovative formulation of fatigue damage at a smaller time scale is of great importance to overcome the inherent difficulties in existing fatigue analysis methodologies.

Conventional fatigue analysis is classified into two categories: crack initiation analysis, e.g., \( S-N \) curve based or \( e-N \) curve based, and crack propagation analysis, e.g., \( da/dN \) approximately delta \( K \) curve based. One challenging problem is the clear separation of these two stages is somewhat unclear and depends on particular applications. The authors have demonstrated that a unified approach using crack growth analysis and equivalent initial flaw size concept can be used for both stages and match the final failure time (Lu et al. 2010; Xiang et al. 2010). The proposed study focuses on the new formulation of crack growth analysis and does not cover the equivalent initial flaw size (EIFS) concept. A novel small time scale fatigue formulation has been recently developed by the authors (Lu and Liu 2010), which is fundamentally different with the traditional cycle-based approach. It describes the crack growth kinetics at any arbitrary time on the basis of the geometry at the crack tip. This model is referred as the “small time scale model” in this paper hereafter. One of advantages of this proposed small time scale model is that it can be coupled with structural dynamics because the crack growth rate can be described as a first-order differential equation. This greatly facilitates the fatigue prognosis for structural systems.

The paper is organized as follows. A brief introduction of the small time scale model for material level fatigue crack growth analysis is given first. Next, the proposed small time scale model is coupled with structural dynamics using the state-space model. Following this, two numerical examples, including single degree-of-freedom (SDOF) and multiple degree-of-freedom (MDOF) problems, are used to show the proposed methodology. Detailed derivation and calculation procedures are discussed. The experimental coupon data for high performance steel (HPS) and aluminum alloy from the literature are used to validate the proposed method. Finally, the proposed methodology is applied to a steel-girder bridge for demonstration. Several conclusions are drawn on the basis of the current investigation.
Small Time Scale Crack Growth Model

The small time scale model is on the basis of the incremental crack growth at any arbitrary small time increment. Only a brief introduction is given in this section to show the fundamental basis of the proposed concurrent structural dynamics and material fatigue crack growth analysis framework. Detailed derivation and model validation for the new material fatigue crack growth model is given by (Lu and Liu 2010).

The small time scale model is developed on the basis of the geometric relationship between the crack tip opening displacement (CTOD) and the instantaneous crack growth kinetics. The geometric relationship between CTOD and the instantaneous crack growth kinetics is shown in Fig. 1. The schematic illustration in Fig. 1 is for a through thickness crack in an infinite plate. Only the tip region is shown. The geometric correction factor can be added for the stress intensity factor calculation for different crack configurations [see Eq. (13)]. For the ease of model illustration, the simplest case of the through thickness crack in an infinite plate is used to derive Eqs. (4)–(10), which is also used by Lu and Liu (2010). As shown in Fig. 1, the crack will extend a distance \( da \) after a small time increment \( dt \), and the crack tip will extend from \( O \) to \( O' \). Considering the geometry of crack tips at two time points \( t \) and \( t + dt \), the crack growth rate \( da/dt \) for an infinitesimal crack growth is derived as it is shown in Eq. (4), where \( \theta \) is the crack tip opening angle (CTOA)

\[
d_{a} = 
\begin{aligned}
\cot \theta & = \frac{1}{2} \cdot \delta = C \delta
\end{aligned}
\]  

Eq. (4) assumes the infinitesimal crack growth. The CTOA changes from 90° at the very beginning of crack growth to a very small angle roughly (approximately 4–6°) beyond a certain length until the final failure (Ma et al. 2003; Newman 2003; Sander and Richter 1987). The CTOA in the small time scale model is assumed to be a function of applied load and material properties. The CTOA is expressed as Eq. (5).

\[
\theta = \frac{\pi}{2} - \frac{\pi \cdot \Delta K - \Delta K_{TH}}{2 \cdot \lambda \cdot K_{TH}}
\]  

\( \Delta K_{TH} \) = intrinsic threshold stress intensity factor; and \( K_{TH} \) = fracture toughness. It is assumed according to the experimental observations (Newman 2003) that the CTOA is 90° when the applied load is approaching the threshold region, whereas it is 0° when the applied load is approaching the fracture toughness. The CTOA can be approximately expressed as Eqs. (6) and (7) using the plastic zone model proposed by Irwin (Janssen et al. 2004). Eq. (6) is for elastic-perfect-plastic material behavior and ignores the hardening effect (Skorupa 1999)

\[
\delta = \frac{1}{2} \frac{K_{y}^{2}}{E \sigma_{y}} = \lambda \int^{2} a
\]

\[
\lambda = \frac{\pi}{2 E \sigma_{y}}
\]  

where \( E \) = Young’s modulus; and \( \sigma_{y} \) = yield strength. In Eq. (6) \( \sigma \) is the nominal stress. According to Eq. 1, the crack propagated from \( O \) to \( O' \) during a small time increment. The crack length increment is \( da \) and the CTOD increment is \( d\delta \). The CTOD increment \( d\delta \) is expressed as Eq. (8)

\[
d\delta = \lambda(2\sigma a d\sigma + \sigma^{2} da)
\]  

Substituting Eq. (8) into Eq. (4) and dividing on both sides by a small time increment \( dt \), the instantaneous crack growth rate is represented as

\[
\frac{1}{C a \cdot dt} = \frac{2\sigma \cdot d\sigma}{1 - C \lambda a^{2} \cdot dt}
\]  

The proposed methodology describes crack growth rate in terms of time scale instead of cycle according to Eq. (9). The crack length at any arbitrary time can be calculated by direct time integration. The previous discussion is for the case when the crack starts to grow. The crack may not grow during the entire duration of the cyclic loading. A general expression considering the nonuniform crack growth is expressed as

\[
\dot{\sigma} = H(\dot{\sigma}) \cdot H(\sigma - \sigma_{REF} \cdot \frac{2C \lambda a}{1 - C \lambda a^{2}} \cdot \dot{\sigma} \cdot \sigma \cdot a)
\]  

where “\( \cdot \)” = time derivative throughout this paper; \( H \) = Heaviside function; and \( \sigma_{REF} \) = reference applied stress level at which the crack starts to grow. Cracks do not grow during the entire duration of the cyclic loading. For example, a crack does not grow during the unloading path because of the energy principle. Also, a crack only starts to grow when the applied loading is beyond a certain stress level. The crack length needs to be calculated by performing the integration from the lower integration limit, e.g., the time when the reference stress level is reached, to the upper integration limit, e.g., the time when the maximum stress level is reached. The crack closure model is essentially contained in the calculation of reference stress level. In this case, the determination of the reference stress level is critical to perform the integration. The small time scale model has been validated using the existing experimental data for various materials under both constant loading and variable amplitude loadings (Lu and Liu 2010).

There are several advantages for the developed small time scale model: (1) Comparing with Paris’ model, the proposed small time scale model does not need cycle counting, e.g., only using the direct time domain integral, under random variable loadings. Paris’ model only considers the average crack growth per cycle and cannot include the detailed mechanisms within one loading cycle, e.g., nonuniform crack growth kinetics within one cycle as shown in Eq. (10); (2) the stress ratio effect has been included in the small time scale model because the direct stress state instead of stress range is used; and (3) one unique advantage of the small time scale model is that it can be seamlessly coupled with structural analysis and has great potential for concurrent structural dynamic analysis and multilevel, e.g., structural level and material level, fatigue damage prognosis. This capability makes it an ideal model for real-time.
damage analysis and online decision making. Detailed discussion is given subsequently.

**Coupled Structural Dynamics and Fatigue Crack Growth Analysis**

The small time scale fatigue model is time based and can be expressed as a set of first-order time derivative equations [see Eq. (10)]. In most cases, fatigue cyclic loading is caused by structural dynamics. Governing equations for structural dynamics is a set of second-order time derivative equations. Mathematically, both material damage and structural dynamics can be expressed using a coupled state-space model. The general procedure for the proposed methodology is demonstrated in Fig. 2. First, governing equations for the structural dynamics are built and expressed using a state-space model (Hamilton 1993). Following this, the critical spot in the structure is identified, and a crack growth equation is associated with this critical spot. Multiple critical spots can be identified, and each of these critical spots can be associated with a crack growth equation. Next, the crack growth equations and structural dynamics equations are coupled together using the state-space model. The coupled state-space equations are solved using time domain numerical integrations. Material level crack growth and structural level dynamic responses are calculated simultaneously. In Fig. 2, generic functions are used to express the coupled state-space equations, which may or may not be linear.

The previously discussed general procedure is illustrated using two simple numerical examples. The first example is an SDOF problem. The second example is an extension of the developed methodology to an MDOF system. These two simple examples show the detailed simulation procedure before the practical demonstration example is performed.

**Single Degree-of-Freedom Dynamic System**

To demonstrate the concept of coupling the small time scale model with a dynamic system, an SDOF spring-mass system is used for illustration purposes. The spring is assumed to be a plate specimen with a center through crack. The material for this specimen is assumed to be aluminum alloy Al7075-T6. The ideal mass-spring-damper system with mass (1 kg), spring constant assumed to be identical to specimen Young’s modulus, and viscous damper of damping coefficient (5 N × s/m) is subject to an oscillatory force \( F(t) \). The properties of the material and specimen are listed in Table 1 (Porter 1972). The governing equation of the dynamic system (Newman 2003) can be express as

\[
m\ddot{x} + n\dot{x} + kx = F(t)
\]

where \( m \) = mass; \( n \) = damping coefficient; \( k \) = stiffness; and \( x \) = displacement of the mass. The structural dynamics is generally described using a second-order time derivative function [Eq. (11)]. It can also be expressed as two first-order time derivative functions and is named as a state-space model (Hamilton 1993). Detailed discussion about the expression of structural dynamics in its state-space formulation is beyond the scope of this study. Interested readers can find details by Hamilton (1993). The proposed small time scale model is essentially a first-order time derivative function and is better coupled with structural dynamics using the state-space model.

**Table 1. Dimensions and Material Properties of the Specimen**

<table>
<thead>
<tr>
<th>Property name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (MPa)</td>
<td>69,600</td>
</tr>
<tr>
<td>Yielding stress (MPa)</td>
<td>520</td>
</tr>
<tr>
<td>Width of cross-section (mm)</td>
<td>100</td>
</tr>
<tr>
<td>Length of the spring (mm)</td>
<td>1,000</td>
</tr>
<tr>
<td>Initial crack size (mm)</td>
<td>6.35</td>
</tr>
<tr>
<td>( \Delta K_{th} ) (MPa × m( \frac{1}{2} ))</td>
<td>0.8</td>
</tr>
<tr>
<td>( K_c ) (MPa × m( \frac{1}{2} ))</td>
<td>68</td>
</tr>
</tbody>
</table>

Fig. 2. General procedure for concurrent structural dynamic analysis and fatigue prognosis in system level
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= (-k/m)x_1 + (-n/m)x_2 + F(t)/m \\
\dot{x}_3 &= H[f(x_2)]H[g(x_1) - \sigma_{\text{REF}}] \cdot \frac{2\alpha}{1-\varepsilon g(x_1)} f(x_2) g(x_1) x_3
\end{align*}
\]  

The derivative for stress \( \sigma \) in Eq. (10) is represented as a generic function of \( x_2 \), e.g., \( f(x_2) \), and the stress is expressed as a generic function of \( x_1 \), e.g., \( g(x_1) \), which can be easily obtained from elasticity. The first two state-space equations (structural level) are directly coupled with the third equation (fatigue analysis at the material level) in Eq. (12). The stiffness coefficient will depend on the crack length and is directly coupled with the material level damage in the proposed heretical state-space model. If the crack length is small, its effect on the component stiffness is small and the coupling effect is not significant. For large cracks, e.g., large enough to affect the compliance of the component, the coupling effect cannot be ignored.

The crack length is obtained by solving the first-order differential equations in Eq. (12). A numerical integration method, i.e., fourth-order Runge-Kutta algorithm in the current study, is used to solve the first-order differential equations. For illustration purposes, an example of variable block loading is shown in Fig. 3. The block loading consists of 50 cycles of constant loading and 3 cycles of overload loading. Fig. 3(a) shows the external loading applied on the dynamic system, and Fig. 3(b) shows the nominal stress history of the specimen. The crack length evolution in the specimen is shown in Fig. 3(c). As shown in Fig. 3(c), the crack growth rate will decrease after the overload, which is known as the retardation phenomena. It is shown in the developed small time scale model (Lu and Liu 2010) that the crack growth rate under fatigue cyclic loading is controlled by two different types of plastic zones: forward plastic zone during the loading path and reverse plastic zone during the unloading path. The reverse plastic zone after unloading produces a compressive residual stress ahead of the crack tip, which indicates the crack does not grow unless the compressive residual stress is reversed. The crack begins to propagate again when the present forward plastic zone size reaches the previous reverse plastic zone size. An overload produces a much larger reverse plastic zone compared with that under constant amplitude loadings and makes the crack growth rate slow down in the following cycles, i.e., crack growth is retarded. After the crack grows beyond the overload affected region, the retardation effect disappears and the crack growth rate “catches up” to the regular behavior. As is shown in Fig. 3, the dynamic response of the system (the local stress) and the crack growth are obtained concurrently. Therefore, the real-time prognosis for system level is achieved.

**Multiple Degree-of-Freedom Dynamic System**

The SDOF dynamic system shows the basic concept of proposed concurrent analysis framework using the state-space model. In practical situations, a structural system is usually an MDOF system. In this section, the proposed concept is extended to the MDOF system with the help of finite-element discretization. A cantilever beam problem is used as a demonstration example. The beam is shown in Fig. 4(a). The material for the MDOF case is assumed to be aluminum alloy AL2024, as shown in Table 2. The beam has been divided into 10 elements to solve the dynamic response (Riera et al. 2004). At the fixed end of the beam, an edge crack over the entire beam width is assumed. A random force is applied at the free end of the beam, as shown in Fig. 4(a). Only an opening mode under bending is considered in this demonstration example, as

![Fig. 3.](image_url)
shown in Fig. 4(b). The stress intensity factor under this condition is expressed in Eq. (13) (Janssen et al. 2004) where \( a \) = crack length; \( b \) = width of the beam; \( \sigma \) = local stress for the crack, which is represented as a function of displacement; and \( F \) = geometric correction function

\[
K_I = F \times \sigma \sqrt{a}
\]

\[
F = 1.122 - 1.40(a/b) + 7.33(a/b)^2 - 13.08(a/b)^3
+ 14.0(a/b)^4
\]  

(13)

The structural governing equation is expressed in the matrix format as Eq. (14)

\[
[M]_{n}\dot{x} + [N]_{n}\dot{x} + [K]_{n}x = F(t)
\]  

(14)

Following the same procedure for the SDOF analysis, the crack growth rate is the state variable and the displacement and velocity. The outputs of the state-space model are crack length \((y_2)\) and displacements at all DOFs. The output variables can be selected from any state variables from the state-space equations, e.g., velocities and displacement. The coupled state-space model can be expressed as given in Eqs. (15) and (16)

\[
\begin{bmatrix}
\dot{x}_1 \\
\vdots \\
\dot{x}_{20} \\
\dot{y}_1 \\
\vdots \\
\dot{y}_{20} \\
\dot{a}
\end{bmatrix}
= 
\begin{bmatrix}
0 & \cdots & 0 & 1 & \cdots & 0 & 0 \\
0 & \cdots & 0 & 0 & 1 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & 0 & 0 & 0 \\
0 & \cdots & 0 & 0 & 0 & 0 & f(x, v)
\end{bmatrix}
\begin{bmatrix}
x_1 \\
\vdots \\
x_{20} \\
y_1 \\
\vdots \\
y_{20} \\
a
\end{bmatrix}
\]  

(15)

\[
\begin{bmatrix}
y_1 \\
\vdots \\
y_{20} \\
a
\end{bmatrix}
= [f] \times
\begin{bmatrix}
x_1 \\
\vdots \\
x_{20} \\
a
\end{bmatrix}
\]  

(16)

The structural dynamic analysis and fatigue crack growth analyses are performed simultaneously using the coupled state-space model according to Eqs. (15) and (16). One benefit of the proposed method is that the coupling effect can be directly solved. The structural stiffness or compliance depends on the material damage, e.g., crack length in the fatigue crack growth analysis. For example, (Guinea et al. 1998) proposed a compliance equation of cracked beams under bending. The compliance of the cracked beam can be expressed as

\[
C_c(\alpha) = \frac{1}{Eb}
\left[ c_1(\alpha) + \beta c_2(\alpha) + \beta^2 c_3(\alpha) \right]
\]  

(17)

where

\[
\alpha = \frac{a}{D}, \quad \beta = \frac{L}{D}
\]  

(18)

\[
c_1(\alpha) = -0.378\alpha^3 \ln(1 - \alpha) + \alpha^2 \left( \frac{0.29 + 1.39\alpha - 1.6\alpha^2}{1 + 0.54\alpha - 0.84\alpha^2} \right)
\]  

(19)

\[
c_2(\alpha) = 1.1\alpha^3 \ln(1 - \alpha) + \alpha^2 \left( -3.22 - 16.4\alpha + 28.1\alpha^2 - 11.4\alpha^3 \right)
\]  

(20)

\[
c_3(\alpha) = -0.176\alpha^3 \ln(1 - \alpha) + \alpha^2 \left( -2.91 - 4.88\alpha - 0.435\alpha^2 + 0.26\alpha^3 \right)
\]  

(21)

where \( \alpha \) and \( \beta \) = crack-to-depth ratio and span-to-depth ratio, respectively; and \( E = \) Young’s modulus. It is shown that the component compliance is a nonlinear function of the crack length. As the
crack length increases, the stiffness reduces. The reduced stiffness may affect the dynamic response and local loading spectra. Thus, the crack growth kinetics also change. This coupling effect can be directly included in the proposed coupled hierarchical state-space modeling. Results for the cantilever beam example are shown in Fig. 5 using the existing solution from Guinea et al. (1998) for the cracked beam problem. For comparison purposes, numerical predictions with and without coupling effect are shown in Fig. 5. Solid lines and dash lines are responses without and with the considered coupling effect between the crack size and element stiffness, respectively. The displacements for two numerical examples with and without coupling effect are shown in Fig. 5(a). The x-axis is the time in term of number of cycles. The y-axis is the beam end displacement. The initial crack length is 1% of the total height. As is shown in Fig. 5(a), the dynamic responses of the two cases are almost identical until the end to the final life, i.e., the large difference after almost 15.5 kilocycles, in which the crack length is very long. This is because the initial crack length is very small and stiffness reduction can be ignored during most of the fatigue life. Fig. 5(b) shows the crack growth curve of these two examples. The x-axis is the time in term of number of cycles. The y-axis is the crack length. It can be clear seen that the crack growth prediction is almost the same, with slight a difference in the near failure regime. If the fatigue life is the final objective, ignoring the coupling effect will not lead to a large error for very small initial cracks. Another example with a larger initial crack is also performed. The initial crack length is 50% of the beam height. The dynamic response with and without considering the coupling effect is shown in Fig. 5(c), and the predicted crack lengths are shown in Fig. 5(d). Fig. 5(c) shows that the dynamic responses of the two cases, i.e., with and without considering the coupling effect, become different after approximately 20 cycles. Fig. 5(d) shows a large difference for the two cases for the crack growth and life predictions. For these cases, ignoring the coupling effect will lead to a large error in the final life prediction. For complex bridge structures with realistic loadings, future study is required. For example, when the vehicle dynamics and wind effects are considered, the damage behavior might be different because of the complex dynamic effects. If multiple cracks exist in the bridge structure, their interaction and dynamic response change is much more complicated. The current investigation aims to propose a general methodology for the concurrent coupled analysis, and application to realistic large bridge structures is beyond the scope of current study. In the later section of this paper, a preliminary study is shown for a simple bridge structure for demonstration purposes.

Model Validation and Comparison with Experimental Data

The previous discussion focuses on the model development, and this section focuses on the model prediction validation. In this section, two different types of material from the literature are chosen: HPS (Chen et al. 2007) and aluminum alloy Al 7075-T6 (Porter 1972). Both constant loading and block loading with overload have been considered.

Fig. 5. Dynamic system response and crack growth trajectories of the beam problem: (a) displacement responses of the cracked segment with a small initial crack size; (b) crack growth trajectories with a small initial crack size; (c) displacement responses of the cracked segment with a large initial crack size; (d) crack growth trajectories with a large initial crack size.
High Performance Steel

Because of its high yield strength, better weldability, and toughness, HPS is increasingly being used for new bridges (Chen et al. 2007; Miki et al. 2002). The HPS experimental data for the ASTM A709 Grade 70W (485W) plate specimen with edge crack under different stress ratio are used to validate the proposed method. The experimental data used are taken from Chen et al. (2007). The geometry information for the single-edge tension specimen is shown in Fig. 6. Each specimen is precracked and tested under constant load conditions in accordance to the ASTM-specified procedures. A summary of the collected experimental data is listed in Table 3. Four specimens are divided into two groups with $R$ ratio of 0 and 0.5, respectively. The material properties of the plate specimens are listed in Table 4. The $\Delta K_{TH}$ and $K_c$ values are required for crack growth rate calculation in Eq. (5). The $\Delta K_{TH}$ and $K_c$ values are calibrated using the crack growth curve data under one stress ratio (Lu and Liu 2010), and the calibrated values are used for predictions for other data sets. The calibrated values for $\Delta K_{TH}$ and $K_c$ are shown in Table 4.

The predictions of the proposed method and experimental data are shown by the $\Delta K$ versus $da/dN$ plot. The comparisons are shown in Fig. 7. A general satisfactory agreement between the model predictions and experimental observations is observed.

<table>
<thead>
<tr>
<th>Load ratio $R$ ($P_{min}/P_{max}$)</th>
<th>Specimen designation</th>
<th>Maximum testing load $P_{max}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>HPS-1</td>
<td>15.0</td>
</tr>
<tr>
<td></td>
<td>HPS-2</td>
<td>16.0</td>
</tr>
<tr>
<td>0.5</td>
<td>HPS-3</td>
<td>28.0</td>
</tr>
<tr>
<td></td>
<td>HPS-4</td>
<td>26.0</td>
</tr>
</tbody>
</table>

Table 4. Material Properties of the Specimen

<table>
<thead>
<tr>
<th>Property name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (MPa)</td>
<td>201,200</td>
</tr>
<tr>
<td>Yielding stress (MPa)</td>
<td>438</td>
</tr>
<tr>
<td>$\Delta K_{TH}$ (MPa × m$^{1/2}$)</td>
<td>0.7</td>
</tr>
<tr>
<td>$K_c$ (MPa × m$^{1/2}$)</td>
<td>500</td>
</tr>
<tr>
<td>Hardening factor</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Fig. 6. Specimen geometry

Fig. 7. Comparison between prediction and experimental data for HPS: (a) load ratio $R = 0$, maximum testing load 15 kN; (b) load ratio $R = 0$, maximum testing load 16 kN; (c) load ratio $R = 0.5$, maximum testing load 28 kN; (d) load ratio $R = 0.5$, maximum testing load 26 kN
Some differences are observed for the stress ratio $R = 0.5$, and additional investigations for high stress ratios are required. Because no variable amplitude loading data are reported, no comparison is performed.

**Aluminum Alloy Al 7075-T6**

In this section, experimental data on aluminum alloy Al 7075-T6 specimens reported by Porter (1972) are used to validate the proposed structural system state-space model. The material properties, e.g., the Young’s modulus, $\Delta K_{TH}$ and $K_c$, are listed in Table 1. For the HPS case, only constant loading is studied. Variable amplitude loading experimental data are used for Al 7075-T6. Variable amplitude loadings containing a different number of overload cycles are shown in Fig. 8 where $n$ is number of cycle for constant loading and $m$ is number of overload loading; both $n$ and $m$ are indicated in Fig. 9. It becomes constant loading for the $m = 0$ case. As discussed in the SDOF example, the crack growth rate will slow down in the following cycle after overload occurs because of the larger reverse plastic zone induced by the overload. One constant loading ($m = 0; n = 50$) and three variable amplitude loading with overload cases ($m = 1; m = 6; m = 50$) have been studied. The comparison between prediction results and experimental data has been illustrated in Fig. 9. A good agreement is observed for the aluminum alloy Al 7075-T6.

Extensive model validation for other metallic materials under both constant and variable amplitude loadings can be found in the paper by Lu and Liu (2010).

**Demonstration Example for Bridge**

The proposed methodology is demonstrated in this section for the concurrent structural dynamic analysis and fatigue prognosis of a steel bridge. The demonstration bridge used here is a two-span continuous steel I-girder bridge coming from the third edition of Load and Resistance Factor released by the National Steel bridge Alliance (2004). The bridge cross section under consideration is given subsequently in Fig. 10.
Four girders are spaced at 3.05 and 1.06 m overhangs, and the roadway is centered over the girders. The girder elevation for the bridge is shown in Fig. 11(a). The section transitions are provided at 30% of the span length (8.23 m) from the interior pier. The girder extending from 0 to 8.23 m on each side of the pier is referred to as “interior girder”, and the girder extending from the abutment to 19.20 m is referred to as “exterior girders”, as shown in Figs. 11(b) and 11(c), respectively. The same girder design for both interior and exterior girders is used in this paper. The structural steel is ASTM A709, Grade 50W. The concrete slab is reinforced with normal Grade 60 reinforcing steel. The detail properties for the steel girders are listed in Table 5 (National Steel Bridge Alliance 2004).

Table 5. Material Properties of the Steel Girder

<table>
<thead>
<tr>
<th>Property name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component and attachment dead load (kg/m)</td>
<td>2,333</td>
</tr>
<tr>
<td>Wearing surface dead load (kg/m)</td>
<td>317</td>
</tr>
<tr>
<td>Moment of inertia for the exterior girder (m^4)</td>
<td>0.0066</td>
</tr>
<tr>
<td>Moment of inertia for the interior girder (m^4)</td>
<td>0.0082</td>
</tr>
<tr>
<td>Young’s modulus (MPa)</td>
<td>206,840</td>
</tr>
<tr>
<td>Yielding strength (MPa)</td>
<td>342</td>
</tr>
<tr>
<td>ΔK_{TH} (MPa × m^{1/2})</td>
<td>0.8</td>
</tr>
<tr>
<td>K_{c} (MPa × m^{1/2})</td>
<td>198</td>
</tr>
</tbody>
</table>

The weight is equally distributed to all girders.
Field splice joints are reported to have great stress concentration (Leander et al. 2010; Li et al.; Miki et al. 2003; White et al. Minor and Derucher 1992). As is shown in Fig. 12, an initial crack because of defects is assumed at the bottom flange (initial crack size \(a = 3\) mm), which is also the transition area between the interior girder and the exterior girder. The crack will first go through the bottom flange and extend to the web finally. The dimension of the bottom flange is shown in Fig. 12. For the bottom flange, the crack is approximately under uniform uniaxial loading. A simplified crack model is used, as shown in Fig. 13. This simplified crack model only demonstrates the proposed methodology. Other crack configurations, such as a surface semielliptical crack, can be easily used in the proposed study by changing the geometric correction function [see Eqs. (13) and (22)]. The stress intensity factor for this loading case (Fig. 13) can be expressed as Eq. (22) (Janssen et al. 2004).

\[
K_1 = F \times \sigma \sqrt{a}
\]

\[
F = 1.12 - 0.231 \left(\frac{a}{b}\right) + 10.55 \left(\frac{a}{b}\right)^2 - 21.72 \left(\frac{a}{b}\right)^3 + 30.39 \left(\frac{a}{b}\right)^4
\]  
(22)

In this demonstration example, the bridge is simplified to a two-span continuous beam model. The bridge is divided into 12 segments to solve the dynamic response. The sinusoidal force applied is shown in Fig. 14(a). It is worth mentioning that the sinusoidal force is only for demonstration purposes for the proposed fundamental methodology. The research for a more realistic case, including live load and the influence line, is currently undergoing. In addition, application with complex large span bridges needs additional study. The dynamic response for this simplified beam-like bridge, i.e., the local stress shown in Fig. 14(b) and the fatigue crack growth shown in Figs. 14(c) and 14(d), is calculated concurrently. For illustration purposes the first 10-s stress history is shown in Fig. 14(b). Fig. 14(c) is the crack growth during a short time duration, and Fig. 14(d) is the crack growth during a long time duration expressed in kilocycles in terms of classical fatigue analysis results. It is seen that the crack growth in a short time duration shows a “zig-zag” pattern, which may be because of small dynamic loading cycles. In the classical fatigue analysis, the crack growth because of dynamic small cyclic loadings is usually ignored, which might lead to a nonconservative prediction. In addition, classical fatigue analysis under the variable loading, as shown in Fig. 14(b), requires cycle counting before the fatigue damage analysis. It is not possible to perform the concurrent analysis because of the limitations of the classical fatigue formulation, i.e., cycle based.

The focus of the proposed study is to demonstrate the coupled material fatigue damage and structural dynamics using simplified structural models and sinusoidal force. For many bridge fatigue analyses, the fatigue cyclic loading is calculated using the influence line or surface from moving loads. Dynamic behaviors are usually ignored if the effect is not large. The proposed general methodology can handle both cases, i.e., static and dynamic analysis. If the dynamic response is ignored, i.e., acceleration and velocity terms are dropped off, the proposed coupled state-space model can be still used. In general, all bridge fatigue loadings are caused by dynamic responses although a simplified moving vehicle load is a good approximation for some bridges. If vehicle dynamics and wind effects cannot be ignored, dynamic response has to be considered. The main objective of the proposed study is to show a general methodology in which the dynamic behavior could be directly coupled with the fatigue damage analysis.
Conclusions

In this study, a new methodology for the real-time structural damage prognosis for a bridge is proposed. A small time scale fatigue crack model at the material level is directly coupled with the structural dynamics using the state-space model. Compared with the traditional cycle-based fatigue crack growth model, it is capable of computing the crack length at any arbitrary time without cycle counting. This advantage makes it possible to integrate the model with a dynamic structural system for concurrent analysis. Several examples are given to demonstrate the proposed concept, and the model predictions are validated using experimental data for HPS and aluminum alloy Al 7075-T6. The proposed methodology is demonstrated using a bridge, and the results for the real bridge case show the capability of the proposed methodology for real-time fatigue prognosis. Several conclusions are drawn on the basis of the proposed study:

1. Concurrent structural dynamic analysis and material fatigue crack growth analysis can be performed using the proposed coupled hierarchical state-space model;
2. The proposed model shows a satisfactory accuracy for the high performance steel used for steel bridges and aluminum alloy Al 7075-T6; and
3. The proposed model can include the coupling effect between structural dynamics and material damage accumulation. This is a more general analysis methodology for structural level damage analysis because the structural damage and structural dynamic response are usually correlated. The coupling effect between dynamic response and material crack growth depends on the initial crack length. In the current investigation (loading and structural configuration), the coupling effect can be ignored for very small initial cracks. Further study is requested for other cases.

The proposed method has several advantages compared with to traditional fatigue damage prognosis methods. First, the fatigue prognosis problem can be directly integrated with the health monitoring system using the proposed hierarchical state-space formulation. The measured dynamic response of a bridge can be used to get the fatigue life without cyclic counting. The real-time fatigue damage prognosis and health management can be achieved. In addition, the fatigue crack model used in this paper does not suffer from the cycle-counting requirements and stress ratio effects compared with the cycle-based fatigue prognosis approach, which significantly reduces the experimental calibration work. Current work focuses on the deterministic analysis. Stochastic analysis to address huge uncertainties associated with fatigue damage needs further study. This paper aims to describe a general methodology to illustrate the basic concept of concurrent structural dynamic analysis and fatigue damage prognosis. Practical applications, including vehicle dynamics and health monitoring data, are currently ongoing.

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References


